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II. **Solution by J. M. COLAW, A. M., Monterey, Va.; W. F. BRADBURY, A. M., Head Master Cambridge Latin School, Cambridge, Mass.; M. A. GRUBER, A. M., War Department, Washington, D. C.; WALTER HUGH DRANE, Graduate Student, Harvard University; and J. O. MAHONEY, B. E., M. Sc., Master of Mathematics and Science, Carthage Graded and High School, Carthage, Texas.**

If p is his principal on July 1st, then $\frac{p(100+m)}{100}$=his money on Aug. 1st.

Also, $\frac{p(100+m)}{100}-\frac{pn(100+m)}{10000}$, or $\frac{p(100-n)(100+m)}{10000}$=his money on September 1st.

$$\therefore \frac{p(100-n)(100+m)}{10000}\div p=\frac{(100-n)(100+m)}{10000},=\left[\frac{(100-n)(100+m)}{100}\right]\%.$$

Substituting numerical values for m and n, respectively, we get $\frac{98\times 103}{10000}=1.0094,=100.94\%$.

The result is independent of the value of p.

Also solved by *G. B. M. ZERR.*

ALGEBRA.

88. **Proposed by E. S. LOOMIS, Ph. D., Professor of Mathematics in Cleveland West High School, Berea, O.**

(1) The Indemnity Savings and Loan Company made two loans of $1000 each to "A", one of its borrowers, under the following terms: In the first loan "A" agrees to cancel the $1000 by making 120 payments of $13.50, the first payment to be considered as made on the first of the month in which the loan is made, and the 119 subsequent payments to be made on the first of each subsequent month; in the second loan "A" agrees to cancel the $1000 by making 120 payments of $13.50, the first payment being made on the first of the month following the loan, and the 119 subsequent payments being made on first of the subsequent months. Does the Company sustain any loss in earnings by the second loan over the first loan, and if so how much, and when is (or are) this loss (or these losses) sustained, the rate of interest in each case being considered as 10½% per annum ?

(2) Deduce a formula for each case of proposition (1) by means of which one can find the balance of the loan uncancelled at the end of *any* month, *if* the loan is fully cancelled in 120 payments.

I. **Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.**

(1) Let P=principal, r=rate of interest, n=number of payments, x=each payment. Then it has been shown in several previous problems that

$$x=\frac{Pr(1+r)^n}{(1+r)^n-1}.$$

In the first case "A" gets at once $1000.00—$13,50=$986.50, and pays this amount in 119 monthly installments.

$\therefore P=986.50,\ r=10\tfrac{1}{2}\div 12=0.875\%,\ n=119.$

$$\therefore x=\frac{986.50\times .00875(1.00875)^{119}}{(1.00875)^{119}-1}.$$

x=$13.375 or $0.125 less than $13.50.

In the second case $P=1000$, $r=.00875$, $n=120$.

$\therefore x=\dfrac{1000\times .00875(1.00875)^{120}}{(1.00875)^{120}-1}$.

$\therefore x=\$13.494$.

In the second case the true interest is realized. ∴ 10½ per cent. is charged. In the first case more than 10½ per cent. is charged. ∴ In the first case the borrower by paying \$13.50 pays 12½ cents more each payment than if he paid 10½ per cent. interest per annum.

(2) Pr=interest. $\therefore x-Pr$=principal paid at the first payment.

$P(1+r)-x$=new principal; $Pr(1+r)-rx$=second interest.

$\therefore x(1+r)-Pr(1+r)=(x-Pr)(1+r)$=principal paid at second payment.

Similarly, $(x-Pr)(1+r)^{m-1}$=principal paid at the mth payment.

$\therefore x-Pr$=first term, $1+r$=ratio, m=number of terms.

$\therefore S=\dfrac{(x-Pr)[(1+r)^m-1]}{r}$.

Unpaid principal at the end of m payments is $P-S$.

Let $x=13.50$, $P=1000$, $r=.00875$, $m=60$.

$\therefore S=\$372.72$.

$\therefore P-S=\$1000-\$372.72=\$627.28$ unpaid.

II. Partial Solution by A. H. BELL, Hillsboro, Ill.

(1) The first loan of \$1000 for 10 years at 10½ per cent. amounts to \$2714.06......(1).

The second loan of \$986.50 for $9\frac{11}{12}$ years at 10½ per cent. amounts to \$2655.24......(2).

Let the common form of the amount of an annuity be:

$$\text{Amount annuity}=A\left(\frac{R'^n-1}{r'}\right)\text{......(3)},$$

in which A=\$13.50, n=number of payments, R'=the amount of \$1.00 for one month at 10½ per cent.=\$1.00875, r'=interest for one month at 10½ per cent.=.00875.

∴ The amount of the annuity \$13.50 for 120 months is \$2847.26.....(4); the amount of the annuity \$13.50 for 119 months is \$2809.18 ....(5).

(4)—(1)=\$133.20 is the profit in the first transaction. ....(6);

(5)—(2)=\$153.94 is the profit in the second transaction......(7).

Since the second loan of \$1000 is immediately reduced by the first payment of \$13.50, no comparison can be made with the first loan, for the second actual loan of \$986.50 pays a greater amount of annuity on the dollar and consequently shows a greater amount earned as seen in (7).

III. Solution by FREMONT CRANE, Sand Coulee, Mont.

(*a*) First payment could draw interest for 120 months; last payment could draw interest for 1 month. ∴ average time=60½ months.

(*b*) First payment would draw interest for 119 months; last payment would draw interest for 0 months. ∴ average time is 59½ months.

∴ (*a*) would be the better investment.

$120\times\$13.50=\$1620$; $\$1620\times.10\frac{1}{2}\times\frac{1}{12}=\$14.175$ = profit of first investment over second.

2. (*a*) Let $\$13.50=p$, $.00875=r$, $\$1000=a$; then $(a-p)$ = amount due first of first month; $(a-p)(1+r)-p$ = amount due first of second month; $[(a-p)(1+r-p)](1+r)-p$ = amount due first of third month; $\{[(a-p)(1+r)-p)](1+r)-p\}(1+r)-p$ = amount due first of fourth month.

∴ for any month, as 10,

$\{[(a-p)(1+r)-p](1+r)-p](1+r)-p\}\ [(a-p)(1+r)-p)(1+r)-p^7(1+r)-p]$.

2. $a(1+r)-p$ at end of first month; $[a(1+r)-p](1+r)-p$ at end of second month; $\{[a(1+r)-p](1+r)-p\}(1+r)-p$ at end of third month. At end of tenth month, $\left\{\{[a(1+r)-p](1+r)-p\}^8(1+r)-p\right\}$.

The solution by Dr. Zerr seems to us to be a proper disposition of the problem, but we publish the other solutions for comparison.

Reply to Note of Dr. Drummond on No. 78.—Its one *real* point is, that my solution, on page 43, line 5, omits to denote $-xy=-6$, as $-x_1y_1=-6$, (which please correct). Thus result $x^4-8x^2+16=0$; $y^4-18y^2+81=0$, whose roots are: $x=2$ to $y=3$; $x_a=-2$ to $y_a=-3$, of xy in (I)(II); and also $x_1=2_1$ to $y_1=3_1$; $x_2=-2_1$ to $y_2=-3_1$ (by last two clauses on page 44), the factors of $-x_1y_1=-6_1$ in parity to $x_{11}y_{11}=35$......(*D*) on page 44. Or if the objector insist,—of $x_1y_1=6_1$ and $x_ay_a=6_a$ reading then $xy=6$ and $x_ay_a=6_a$. No other error is specified or shown. As the equations cited do not occur in my solution,—for his $x^2=36$ is quadratic while $x^2y^2=36$ from (I)(II) is bi-quadratic; I need only say that the roots of his $x^4=1296$ are 6; -6, $6\sqrt{(-1)}$; $-6\sqrt{(-1)}$, not ±6; ∓6. Roots $x^4=16$ are 2; -2; $2\sqrt{(-1)}$; $-2\sqrt{(-1)}$ and not ±2; ∓2 as he mistakes me to mean.

J. M. BOORMAN.

GEOMETRY.

99. **Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio State University, Athens, Ohio.**

Find the locus of the vertices of all right cones which have the same given ellipse as a base.

Solution by the PROPOSER.

Any generator, $x=a'z+\alpha$......(1). $y=b'z+\beta$......(2), must satisfy

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ ......(3),\ \text{and}\ z=0\ ......(4),\ \text{giving}\ \frac{\alpha^2}{a^2}+\frac{\beta^2}{b^2}=1\ ......(5).$$

If the vertex be (x', y', z'), (1) and (2) must be satisfied, and we have $x'=a'z'+\alpha$......(6), $y'=b'z'+\beta$......(7).

Eliminating a', b', α, β from (1), (2), (6), (7),

$$\frac{1}{a^2}(z'x-x'z)^2+\frac{1}{b^2}(z'y-y'z)^2=(z'-z)^2\ .....\ (8),$$